CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total grade—50

Part A consists of 28 questions that will be answered on side 1 of the answer sheet. Following are the directions for Section I, Part A.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = 5x^{\frac{4}{3}}$, then $f'(8) =$

(A) 10 (B) $\frac{40}{3}$ (C) 40 (D) 80 (E) $\frac{160}{3}$

2. $\lim\limits_{x \to \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5}$ is

(A) 0 (B) $\frac{4}{5}$ (C) $\frac{3}{11}$ (D) $\frac{5}{4}$ (E) ∞

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3. If $f(x) = \dfrac{3x^2+x}{3x^2-x}$ then $f'(x)$ is

(A) 1

(B) $\dfrac{6x^2+1}{6x^2-1}$

(C) $\dfrac{-6}{(3x-1)^2}$

(D) $\dfrac{-2x^2}{(x^2-x)^2}$

(E) $\dfrac{36x^3-2x}{(x^2-x)^2}$

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4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\dfrac{8}{7}$ (C) -1 (D) 0 (E) undefined

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5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

(A) $\frac{8 + 2y - 2x}{6y - 2x}$

(B) $\frac{3y - x}{y - x}$

(C) $\frac{2x - 2y}{6y - 2x}$

(D) $\frac{1}{3}$

(E) $\frac{y - x}{3y - x}$

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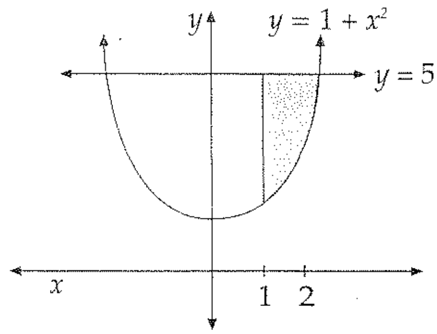

6. Which of the following integrals correctly corresponds to the area of the shaded region in the figure above ?

(A) $\int_1^2 (x^2 - 4)dx$

(B) $\int_1^2 (4 - x^2)dx$

(C) $\int_1^5 (x^2 - 4)dx$

(D) $\int_1^5 (x^2 + 4)dx$

(E) $\int_1^5 (4 - x^2)dx$

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7. If $f(x) = \sec x + \csc x$, then $f'(x) =$

(A) 0
(B) $\sec^2 x + \csc^2 x$
(C) $\csc x - \sec x$
(D) $\sec x \tan x + \csc x \cot x$
(E) $\sec x \tan x - \csc x \cot x$

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8. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2, 4) is

(A) $-4x + y = 20$ (B) $4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$

9. $\int_{-1}^{1} \frac{4}{1+x^2} dx =$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

11. If $f(x) = \frac{5}{x^2+1}$ and $g(x) = 3x$, then $g(f(2)) =$

(A) -3 (B) $\frac{5}{37}$ (C) 3 (D) 5 (E) $\frac{37}{5}$

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12. $\int x\sqrt{5x^2-4}\ dx =$

(A) $\frac{1}{10}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(B) $\frac{1}{15}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(C) $-\frac{1}{5}\left(5x^2-4\right)^{-\frac{1}{2}}+C$

(D) $\frac{20}{3}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(E) $\frac{3}{20}\left(5x^2-4\right)^{\frac{3}{2}}+C$

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13. The slope of the line tangent to the graph of $3x^2 + 5\ln y = 12$ at $(2, 1)$ is

(A) $-\frac{12}{5}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) 12 (E) –7

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14. The equation $y = 2 - 3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of

(A) $\frac{1}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{4}{\pi}$ (D) 8 (E) 2π

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15. If $f(x) = \begin{cases} x^2 + 5 \text{ if } x < 2 \\ 7x - 5 \text{ if } x \geq 2 \end{cases}$, for all real numbers x, which of the following must be true?

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x = 2$.

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, and III

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16. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

(A) 10 (B) 4 (C) 3 (D) -4 (E) -10

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17. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at

(A) $t = 1$
(B) $t = 3$
(C) $t = 5$
(D) $t = 1$ and $t = 5$
(E) $t = 1$ and $t = 3$ and $t = 5$

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18. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

19. $\int \left(e^{3\ln x} + e^{3x}\right)\, dx =$

(A) $3 + \frac{e^{3x}}{3} + C$

(B) $\frac{x^4}{4} + 3e^{3x} + C$

(C) $\frac{e^{x^4}}{4} + 3e^{3x} + C$

(D) $\frac{e^{x^4}}{4} + \frac{e^{3x}}{3} + C$

(E) $\frac{x^4}{4} + \frac{e^{3x}}{3} + C$

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20. If $f(x) = \sqrt{(x^3 + 5x + 121)}\,(x^2 + x + 11)$ then $f'(0) =$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22 + \frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

21. If $f(x) = 5^{3x}$ then $f'(x) =$

(A) $5^{3x}(\ln 125)$ (B) $\frac{5^{3x}}{3 \ln 5}$ (C) $3(5^{2x})$ (D) $3(5^{3x})$ (E) $3x(5^{3x-1})$

22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_1^8 \left(x^2+1\right)^3 dx$

(B) $\pi\int_1^8 \left(x^2+1\right)^6 dx$

(C) $\pi\int_0^1 \left(x^2+1\right)^3 dx$

(D) $\pi\int_0^1 \left(x^2+1\right)^6 dx$

(E) $2\pi\int_0^1 \left(x^2+1\right)^6 dx$

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23. $\lim_{x\to 0} 4\,\frac{\sin x\cos x - \sin x}{x^2} =$

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

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24. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18 (B) $\sqrt{66}$ (C) 58 (D) $\sqrt{74}$ (E) $\sqrt{58}$

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25. $\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\frac{x}{3}\right)+C$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

(C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right)+C$

(D) $\frac{1}{3}\tan^{-1}(x)+C$

(E) $\frac{1}{9}\tan^{-1}(x)+C$

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26. If $f(x) = \cos^3(x+1)$ then $f'(\pi) =$

(A) $-3\cos^2(\pi+1)\sin(\pi+1)$

(B) $3\cos^2(\pi+1)$

(C) $3\cos^2(\pi+1)\sin(\pi+1)$

(D) $3\pi\cos^2(\pi+1)$

(E) 0

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27. $\int x\sqrt{x+3}\,dx =$

(A) $\frac{2}{3}(x)^{\frac{3}{2}} + 6(x)^{\frac{1}{2}} + C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3} + C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2} + C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3} + C$

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28. If $f(x)=\ln(\ln(1-x))$, then $f'(x)=$

(A) $-\dfrac{1}{\ln(1-x)}$

(B) $\dfrac{1}{(1-x)\ln(1-x)}$

(C) $\dfrac{1}{(1-x)^2}$

(D) $-\dfrac{1}{(1-x)\ln(1-x)}$

(E) $-\dfrac{1}{\ln(1-x)^2}$

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Part B consists of 17 questions that will be answered on side 2 of the answer sheet. Following are the directions for Section I, Part B.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING SIDE 2 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 29–45.

YOU MAY NOT RETURN TO SIDE 1 OF THE ANSWER SHEET

In this test:

(1) The ***exact*** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Note: Question numbers with an asterisk (*) indicate a graphing calculator-active question.

29. $\int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{-\frac{\pi}{4}}^{0} \cos x\, dx =$

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

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30. Boats A and B leave the same place at the same time. Boat A heads due North at 12 km/hr. Boat B heads due East at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?

(A) 21.63 (B) 31.20 (C) 75.00 (D) 9.84 (E) 54.08

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31. $\lim_{h \to 0} \dfrac{\tan\left(\dfrac{\pi}{6}+h\right)-\tan\left(\dfrac{\pi}{6}\right)}{h} =$

(A) $\dfrac{\sqrt{3}}{3}$ (B) $\dfrac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\dfrac{3}{4}$

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32. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$

(A) $A + B$ (B) $A - B$ (C) 0 (D) $B - A$ (E) 20

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33. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

(A) 59 (B) $\dfrac{1}{59}$ (C) $\dfrac{1}{10}$ (D) 11 (E) $\dfrac{1}{11}$

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34. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at

(A) (0.46, 2.87) (B) (0.46, 0) (C) (2.87, –4.06) (D) (4.06, 2.87) (E) (1.66, –0.59)

35. The volume generated by revolving about the y-axis the region enclosed by the graphs $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \le x \le 2$, is

(A) -8π (B) 4π (C) 8π (D) 24π (E) 48π

36. The average value of the function $f(x) = \ln^2 x$ on the interval [2, 4] is

(A) –1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636

37. $\frac{d}{dx}\int_0^{3x} \cos(t)dt =$

(A) $\sin 3x$ (B) $-3\sin 3x$ (C) $\cos 3x$ (D) $3\sin 3x$ (E) $3\cos 3x$

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38. If the definite integral $\int_1^3 (x^2+1)dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

(A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

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39. The radius of a sphere is increasing at a rate proportional to its radius. If the radius is 4 initially, and the radius is 10 after two seconds, what will the radius be after three seconds?

(A) 62.50 (B) 13.00 (C) 15.81 (D) 16.00 (E) 25.00

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40. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

(A) 4213.973 (B) 1261.669 (C) 1256.637 (D) 25.233 (E) 25.133

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41. $\int \ln 2x \; dx =$

(A) $\frac{\ln 2x}{x} + C$

(B) $\frac{\ln 2x}{2x} + C$

(C) $x \ln x - x + C$

(D) $x \ln 2x - x + C$

(E) $2x \ln 2x - 2x + C$

42. If the function $f(x)$ is continuous and differentiable $= \begin{cases} ax^3 - 6x; & \text{if } x \le 1 \\ bx^2 + 4; & x > 1 \end{cases}$ then $a =$

(A) 0 (B) 1 (C) −14 (D) −24 (E) 26

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t)=7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

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45. $\int \tan^6 x \sec^2 x\,dx =$

(A) $\dfrac{\tan^7}{7}+C$

(B) $\dfrac{\tan^7 x}{7}+\dfrac{\sec^3 x}{3}+C$

(C) $\dfrac{\tan^7 x \sec^3 x}{21}+C$

(D) $7\tan^7 x+C$

(E) $\dfrac{2}{7}\tan^7 x \sec x+C$

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR NUMBER CORRECTLY IN SECTION C OF THE ANSWER SHEET

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three decimal places.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Consider the equation $x^2 - 2xy + 4y^2 = 64$.

   (a) Write an expression for the slope of the curve at any point (x, y).

   (b) Find the equation of the tangent lines to the curve at the point $x = 2$.

   (c) Find $\frac{d^2y}{dx^2}$ at $(0, 4)$.

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2. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

   (a) Write an expression for the velocity of the particle $v(t)$.

   (b) At what values of t does the particle change direction?

   (c) Write an expression for the position $x(t)$ of the particle.

   (d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

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3. Let R be the region enclosed by the graphs of $y = 2\ln x$ and $y = \frac{x}{2}$, and the lines $x = 2$ and $x = 8$.

(a) Find the area of R.

(b) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x-axis.

(c) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the line $x = -1$.

SECTION B

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4. Water is draining at the rate of $48\pi\text{ft}^3$/minute from a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

(a) Find an expression for the volume of water in the tank in terms of its radius.

(b) At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?

(c) How fast is the height of the water in the tank dropping at the instant that the radius is 16 feet?

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5. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

(a) Find an equation of the line tangent to the graph at (2,17).

(b) Find the x- and y-coordinates of the relative maxima and relative minima. Verify your answer.

(c) Find the x- and y-coordinates of the points of inflection. Verify your answer.

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6. Let $\int_0^x \left[\cos\left(\frac{t}{2}\right) + \left(\frac{3}{2}\right)\right] dt$ on the closed interval $[0, 4\pi]$.

(a) Approximate $F(2\pi)$ using four inscribed rectangles.

(b) Find $F'(2\pi)$.

(c) Find the average value of $F'(x)$ on the interval $[0, 4\pi]$.

END OF EXAMINATION